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Retardation Plates

Application Note

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How to select the right retardation plate for your particular application

You need to control the state of polarization of your light beam? You need to change the direction of polarization of your beam? Or you need to change from linear polarization to circular polarization or vice versa? Or you need to create any other specific state of polarization?

If you must perform any – or any combination – of these tasks, then you will have to use (apart from polarizers) retardation plates; sometimes retardation plates are just called wave plates or retarders. The purpose of a retardation plate is to introduce a specific phase difference or retardation between the two orthogonal components of the electromagnetic light field.

Before discussing the different types of retardation plates, let us start with some general remarks. Retarders are usually made from birefringent materials. These exhibit birefringence: Differently polarized light beams experience different indices of refraction and consequently light polarized in one direction is retarded (or advanced) with respect to light polarized in the other direction when traversing a plate made from such material. The amount of retardation (also called retardance) is determined by the amount of birefringence Δn and the thickness d of the plate.

Retardation can be expressed/measured in various units, and unfortunately all these units are commonly used in industry and science as well:

Retardation	Formula
expressed in unit of length (usually nm)	$\Delta d = \Delta n \times d$
expressed as a fraction or multiple of wavelength	$\Delta\varphi = \frac{\Delta n \times d}{\lambda}$
expressed in radians	$\Delta\varphi = 2\pi \times \frac{\Delta n \times d}{\lambda}$
expressed in degrees	$\Delta\varphi = 360° \times \frac{\Delta n \times d}{\lambda}$

For special applications, additional units are sometimes used, such as time (fs) in ultrafast applications.
For most materials, Δn is essentially independent of wavelength. Therefore, $\Delta d = \Delta n \times d$ is (almost) constant: It determines the pathlength difference produced by the plate and is typically measured in nanometers. In most applications, not the pathlength difference, but the phase difference $\Delta\varphi$, expressed as a fraction of wavelength or in angular units, is of importance. A quarter-wave plate, for example, is a plate that generates a retardation equivalent to ¼ of the wavelength or $\pi/2$ (radians) or 90°; when the plate introduces a retardation of ½ of the wavelength or π (radians) or 180° it is usually called a half-wave plate. These two types are the most commonly used retardation plates; for other retardation values, there is no common terminology. If, however, a particular waveplate is a quarter-wave plate or not, not only depends on the plate itself but evidently also on the wavelength where the plate is used.

Retardation plates are available in various materials, and they come in three basic types:

1. Multiple order plates: They create a large phase difference, several times the wavelength plus a fraction of the wavelength, but only the fraction is of importance in most applications.
2. Quasi-zero order plates: They consist of a pair of almost identical multiple order plates with crystal axes orientated orthogonally, such that only the difference in phase retardation is effective.
3. True-zero order plates: They are extremely thin, such that they create a phase difference which is truly only a fraction of the wavelength.

Before selecting the material, the appropriate type should be determined. If your light beam

- is directionally stable,
- is well collimated,
- has narrow spectral bandwidth,
- travels in a thermally stable environment,
- has low intensity, both spatially and temporally,

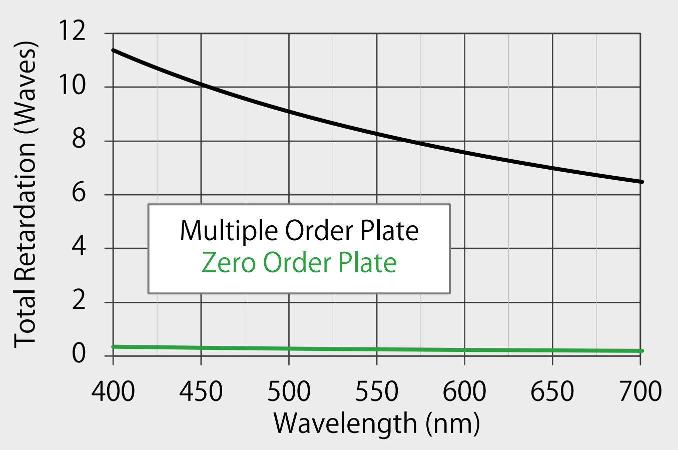
then the type does not really matter, and you should make your selection mainly based on price and other optical properties which are important for all kinds of optical components (apart from other considerations such as lead time, reliability etc. which you apply in all purchasing decisions).

If any of the above conditions is not fulfilled, then the decision about the appropriate type will be more demanding. In the following analysis, the impact of the above conditions on the performance of the three types of wave plates is examined. Retardation precision is shown for the following examples which are paradigmatic for commercially available retardation plates:

1. **Multiple order plate:** Usually a single crystalline quartz plate with such a thickness that the plate can easily be produced by the manufacturer and handled by the user. Many suppliers do not specify the thickness of their multiple order plates at all; some specify a thickness of approximately 0.5 mm. We consider this as the typical thickness.
2. **Quasi-zero order plate:** Usually a pair of quartz plates cemented, optically contacted, or air spaced. Even less suppliers specify the thickness of their quasi-zero order plates, some say 1.5⋯2.5 mm. We consider 2 mm as the typical thickness.
3. **True-zero order plate:** Mica or polymer, bare or cemented between glass flats, thickness corresponding to true-zero order condition.

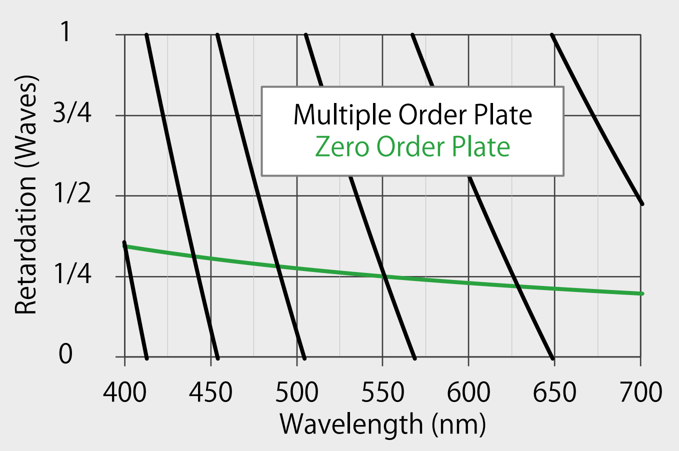
In our discussion, we concentrate on the visible wavelength range, but results are comparable in the UV and in the IR. **All retardation plates are considered to have a good antireflective coating**; the behavior of retardation plates without AR coating can be quite confusing.

Let us first discuss the **spectral behavior** of the different types of retardation plates. Initially, we compare the total retardation (without any "reduction to zero order") of a typical multiple order plate made of quartz to the retardation of a zero order plate, neglecting any dispersion of birefringence, and we take quarter-wave plates as an example:



Both curves are simple hyperbolas, and the curve for the multiple order plate is simply the 33rd multiple of the curve for the zero order plate (8.25 waves @ 550 nm vs. 0.25 waves). **The spectral behavior of quasi-zero order plates is exactly the same as that of s true zero order plates.**

As mentioned above, only the so-called "retardation reduced to zero order" is of importance in most applications, meaning only the fractional part of the total retardation. Then the comparison looks like this:



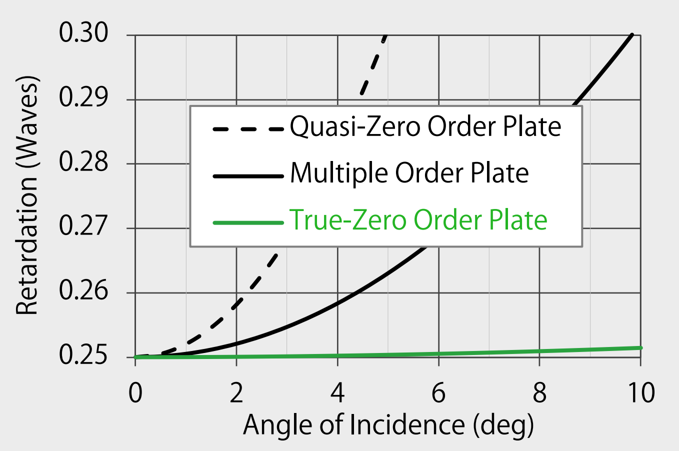
The curves for other design retardations will look very similar.

What can be learned from these diagrams?

- Zero order plates can be used over a fairly wide wavelength range, no matter if they are true-zero order plates or quasi-zero order plates.
- In contrast, the retardation of multiple order plates, measured in angular units or waves, varies rapidly with wavelength – so much, that even the character of the plate (quarter-wave, half-wave ⋯) may be completely lost or reversed over fairly small wavelength ranges.

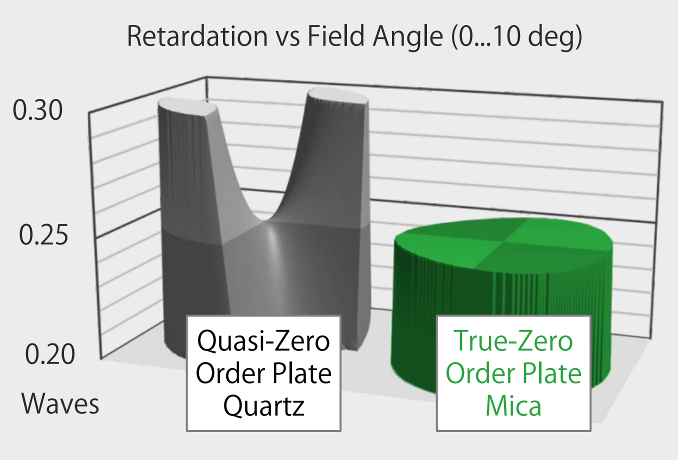
Considering wavelength dependence, a zero order plate is always better than a multiple order plate.

Let us now discuss the **angular behavior** of the different types of retardation plates, using again a quarter-wave plate as an example. In contrast to the spectral behavior, true-zero order plates and quasi-zero order plates do not exhibit the same behavior under varying angles of incidence. Therefore, retardation versus angle of incidence is shown for all three types:



In fact, the retardation not only depends on the angle of incidence as such, but also on the orientation of the plane of incidence with respect to the crystal axes. There are two orthogonal orientations where the retardation does not depend on angle of incidence, and there are two other orientations (rotated by 45°) where the dependence is maximum; these latter orientations were used for the diagram above.

In addition, the deviation from the design retardation can be positive as well as negative, again depending on the orientation. The retardation under all orientations and angles of incidence (AoI) is depicted in the following diagram:



The diagrams are practically independent of the particular design of the retardation plate, i.e., it does not matter if the plate is a quarter-wave or a half-wave plate or has any other retardation value: The errors will scale proportionately.

What can be learned from these diagrams?

- Retardation plates which are not true zero order plates have a strong dependence of retardation on angle of incidence.
- Quasi-zero order plates even produce twice the error produced by the multiple order plates they are made from. Actually, at 10° angle of incidence a quasi-zero order quarter-wave plate almost becomes a half-wave plate and is, insofar, completely unusable.
- The dependence of retardation on angle of incidence is almost invisible for true-zero order plates and practically negligible for most real-life applications.

Consequently, true-zero order plates should be used whenever the relative orientation of the light beam with respect to the retardation plate is not fully controlled.

This is not just a matter of mechanical instability or similar effects, but also a matter of beam collimation. When the beam is not perfectly collimated, then it contains a variety of ray directions – within the cone of beam divergence. All these ray directions will have different angles of incidence on the wave plate, resulting in different phase retardation for different ray directions. Therefore, an initially completely polarized divergent light beam will become partially depolarized on transmission through a retardation plate. Evidently, the depolarization is not uniform over the angular field, but varies with the individual direction of each light ray.

The following situation may serve for a numerical example: A linearly polarized beam of light with a divergence angle of 10° is transmitted through a quasi-zero order half-wave plate oriented at 45° (such that a collimated beam's polarization direction would be rotated by 90°); when analyzed with a crossed polarizer, approximately 27% of the incident light will be transmitted instead of 0%, and for a multiple order half-wave plate, which is not quite as sensitive to angular deviations, it is still 9%. In other words: **A divergent beam becomes heavily depolarized by retardation plates which are not true-zero order.** In contrast, if a true-zero order plate is used, the depolarization is essentially zero (numerically: 0.0015%; this is better than most polarizers can achieve).

We have seen that directional sensitivity directly translates to loss of polarization for divergent beams. Likewise, wavelength dependence translates to loss of polarization for non-monochromatic beams. **Consequently, a true-zero order plate is always a perfect choice.**

The retardation of any given wave plate also depends on temperature for two reasons:

- The plate thickness varies with temperature.
- The indices of refraction, and consequently the birefringence, depend on temperature.

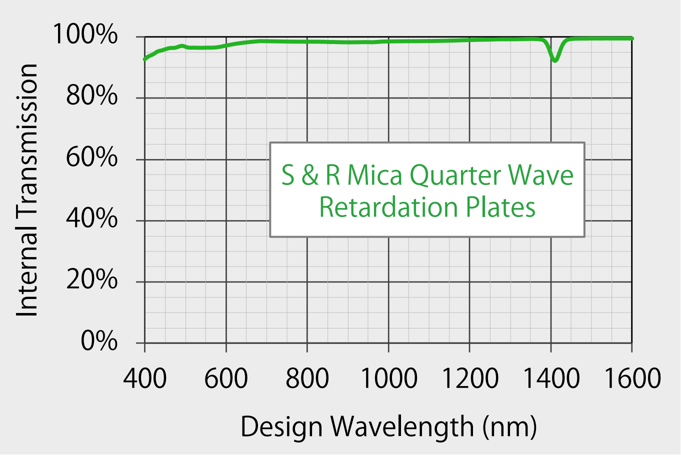
These effects vary with material, but for quartz and mica the temperature dependence is similar. For any given material, the following statement is correct: Quasi- and true-zero order plates exhibit much less temperature dependence than multiple order plates made of the same material. Here an example for quarter-wave plates made of quartz: The multiple order plate will be approximately 33 times more sensitive to temperature variation than the zero order plate.

From the discussions above, the following becomes evident:

- **Whenever the wavelength is likely to change or whenever the spectral bandwidth is not extremely limited, a multiple order plate is not a good choice.**
- **Whenever the beam direction is likely to change or whenever the beam is divergent or convergent, neither a multiple order plate nor a quasi-zero order plate is a good choice.**
- **A true zero order plate is a good technical choice under all conditions that make the other types unsuitable.**

Traditionally, zero order plates were first made of mica because mica is easily available as a natural mineral and was (in the 19th century and before) much more easily handled than other optical crystal materials. In particular, by simply cleaving mica sheets along a natural cleavage plane, wave plates with good quality were obtained without any additional optical manufacturing procedure. Over the years, optical technology progressed, and it became commercially feasible to produce plates from more or less any birefringent material. Furthermore, in the second half of the last century polymers were developed that could be used for phase retardation. At the same time, mica became the material of second choice only. But some companies in the world, for example S & R Optic GmbH of Germany, continued to improve their mica production technology further and further, to the effect that mica is once again an excellent choice for zero order wave plates.

While mica has so many technical advantages compared to most other materials used for retardation plates in the visible and near infrared range, it also has one disadvantage: Mica exhibits noticeable **absorption**. The amount of absorption, however, is often grossly overestimated and can be minimized by careful selection of the raw material. S & R Optic use only mica with extremely low absorption and produce mica retardation plates with very high transmission, shown here by example of quarter-wave plates:



The diagram does not show transmission versus wavelength for any particular thickness of mica, but it directly shows the transmission for quarter-wave retardation plates **at their design wavelength**. Evidently, these values can only be achieved with a proper antireflective coating. Such AR coatings are offered by S&R Optic, even for bare mica retardation plates.

In the following table, the main combinations of technology and optical material are compared from a practical perspective as far as they differ significantly. Bare polymer retarders are not included in this table because – due to their extremely small thickness – they are mechanically highly unstable and not suitable for the vast majority of

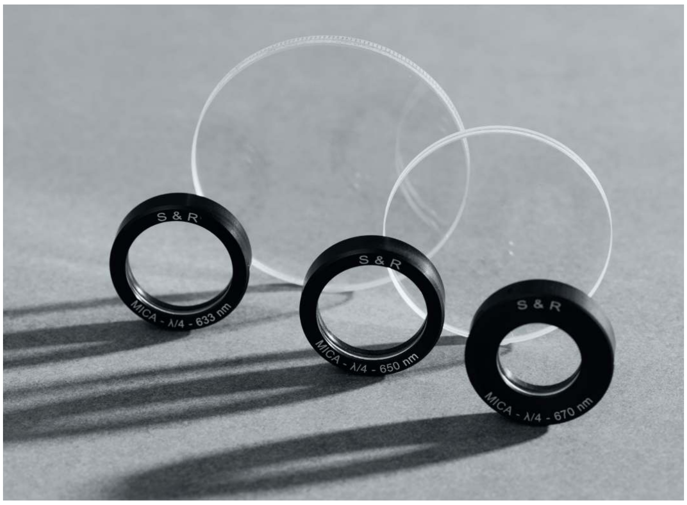
Material	Quartz	Quartz	Quartz	Quartz	Mica	Mica	Polymer
Type	Multiple order	Quasi-zero order	Quasi-zero order	Quasi-zero order	True-zero order	True-zero order	True-zero order
Mounting	Bare	Cemented	Optically contacted	Air spaced	Cemented	Bare	Cemented
Useable wavelength range (nm)	193 – 2000	400 – 2000	193 – 2000	193 – 2000	400 – 1550	350 – 1550	400 - 1550
Practical max. diameter	50 mm	50 mm	50 mm	50 mm	200 mm	200 mm	200 mm
Typical absorption @ 500 nm	negligible	slight	negligible	negligible	few percent	few percent	< 1%
Damage threshold @ 1064 nm, pulsed	10 J/cm^2	0.5 J/cm^2	10 J/cm^2	10 J/cm^2	0.5 J/cm^2	10 J/cm^2	4 J/cm^2
Damage threshold @ VIS - NIR, cw	10 MW/cm^2	1.5 kW/cm^2	1 MW/cm^2	1 MW/cm^2	0.5 kW/cm^2	0.5 kW/cm^2	0.5 kW/cm^2
Homogeneity of retardation over aperture (typ. 25 mm)	$\lambda/500$	$\lambda/300$	$\lambda/500$	$\lambda/500$	$\lambda/300$	$\lambda/300$	$\lambda/100$
Precision of retardation	$\lambda/300$	$\lambda/200$	$\lambda/300$	$\lambda/300$	$\lambda/200$	$\lambda/200$	$\lambda/100$
Temperature dependence of retardation per degree C	0.3 %	negligible	negligible	negligible	negligible	negligible	0.04 %
Price @ low quantity	high	high	high	very high	low	moderate	high
Price @ medium quantity	low	moderate	moderate	high	very low	low	moderate
Price @ high quantity	low	moderate	moderate	moderate	very low	very low	low

The table clearly demonstrates the advantages as well as the disadvantages of the various types of retardation plates. A clear advantage of quartz is the usability in the UV and the very high damage threshold, especially the continuous wave damage threshold which results from the low absorption of quartz. On the other hand, quartz is usually not the material of first choice, particularly due to the pricing which can be prohibitive unless a multiple order plate can really be used in the application.

The absorption, and the associated lower cw damage threshold, of polymer and mica are essentially the only technical disadvantages of these materials in relation to quartz. Therefore, usage of these materials is generally advisable unless the particular application clearly demands one of the positive technical features of quartz. And unless the lower absorption in polymers justifies a considerably higher price, **true-zero order plates made of mica are the best choice** for a vast majority of applications, especially when price is an important consideration. **For mica, "low price" generally even coincides with "good availability"**: Well established producers of mica retarders, such as S & R Optic GmbH, usually have a great variety of bare mica sheets in single nanometer gradation available on stock, so that retardation plates for almost any wavelength in the useable range can be made by just assembling an already existing mica sheet.

In conclusion:

- True-zero order plates are always the best choice under technical considerations.
- Quasi-zero order plates behave well under variation of wavelength and temperature but behave even worse than multiple order plates under variation of beam direction.
- Multiple order plates are only useful when all optical and environmental parameters are well controlled.
- Cemented polymer retarders are a good choice technically but are usually very expensive.
- Mica retarders are generally the preferred choice from a technical point of view, and typically they are more easily available and come at a lower price than other retarders.



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